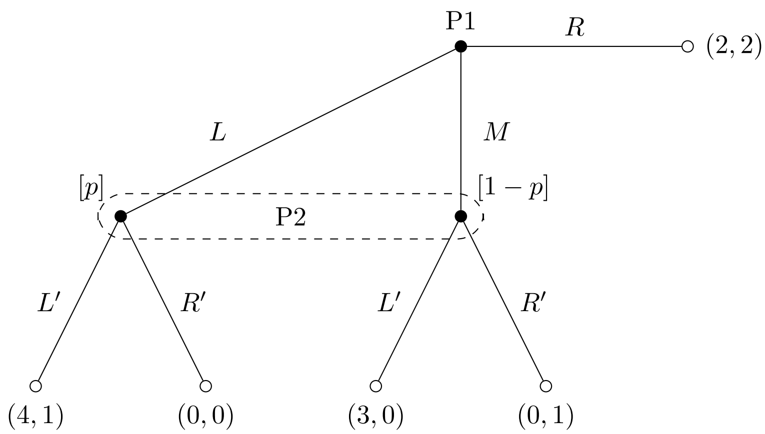
# Game Theory
# Assignment 4 Solution

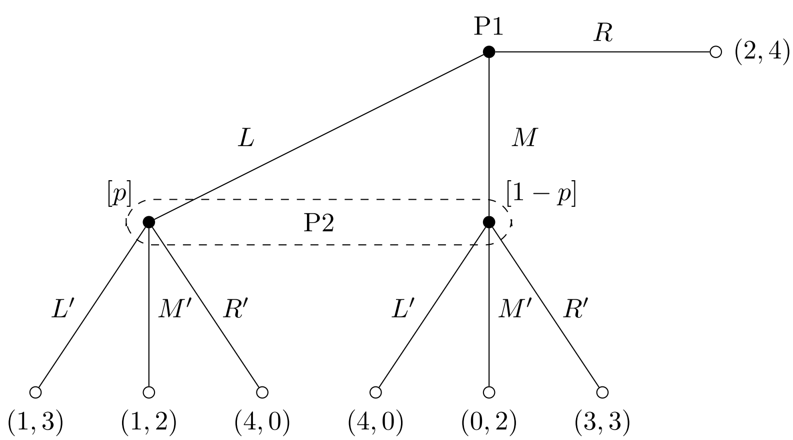
注：此答案步骤较简略，仅供参考。

**Question 1: Gibbons 4.1** In the following extensive-form games, derive the normal-form game and find all the pure-strategy Nash, subgame perfect, and perfect Bayesian equilibria.

a.




b.



**Solution**

a. (i) Normal-form game:

Player 2

| Player 1 | | L' | R' |
|---|---|---|---|
| | L | $(4,1)$ | $(0,0)$ |
| | M | $(3,0)$ | $(0,1)$ |
| | R | $(2,2)$ | $(2,2)$ |

(ii) Pure strategy NE: $(L, L')$ and $(R, R')$

(iii) SPE: $(L, L')$ and $(R, R')$

(iv) PBE: $(L, L', p = 1)$ and $(R, R', p)$ for any $p \leq 1/2$.

b. Normal-form game:

Player 2

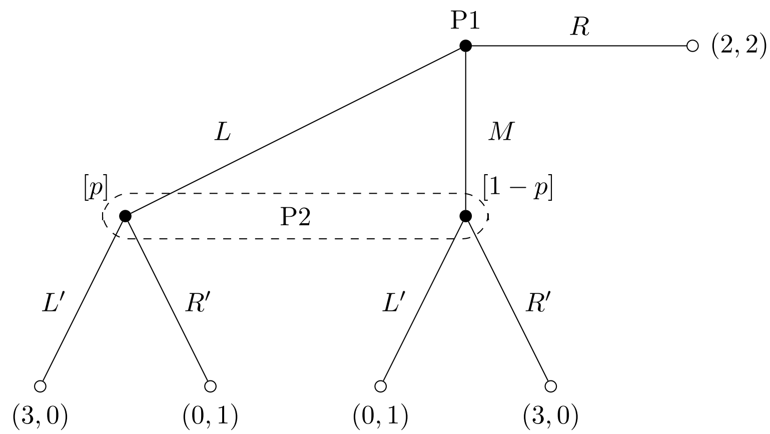
| Player 1 | | L' | M' | R' |
|---|---|---|---|---|
| | L | $(1,3)$ | $(1,2)$ | $(4,0)$ |
| | M | $(4,0)$ | $(0,2)$ | $(3,3)$ |
| | R | $(2,4)$ | $(2,4)$ | $(2,4)$ |

c. Pure strategy NE: $(R, M')$

d. SPE: $(R, M')$

e. PBE: $(R, M', p)$ for any $1/3 \leq p \leq 2/3$.

**Question 2: Gibbons 4.2** Show that there does not exist a pure-strategy perfect Bayesian equilibrium in the following extensive-form game. What is the mixed-strategy perfect Bayesian equilibrium?




**Solution** Let $p$ denote player 2's belief that $L$ has been chosen when the game reaches his/her information set.

1. $L'$ is optimal for player 2 if $p \leq 1/2$. If player 2 chooses $L'$, player 1's best response is $L$. Since $L$ is on the equilibrium path of $(L, L')$, Bayes' rule implies that $p = 1$. This contradicts with $p \leq 1/2$.
2. $R'$ is optimal for player 2 if $p \geq 1/2$. If player 2 chooses $R'$, player 1's best response is $M$. Since $M$ is on the equilibrium path of $(M, R')$, Bayes' rule implies that $p = 0$. This contradicts with $p \geq 1/2$.

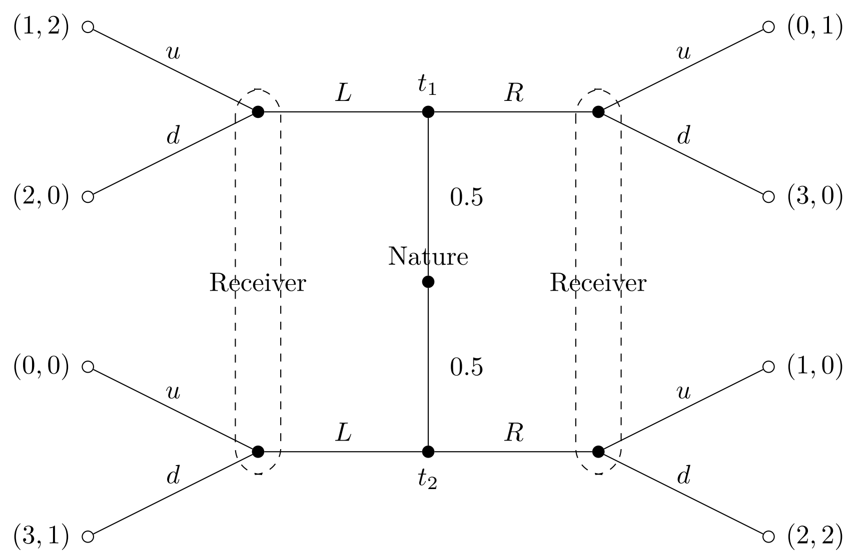
It is optimal for player 2 to mix if $p = 1/2$. Let player 2's mixed strategy be $(q, 1-q)$.

1. Suppose $L$ and $M$ is on the equilibrium path. Then $q = 1/2$. However, when this is the case, player 1's payoff from $L$ or $M$ is $3/2$, which is less than his/her payoff from $R$. We reach a contradiction.
2. Suppose $L$ and $M$ is off the equilibrium path. Then we need to ensure that it is player 1's best response to choose $R$. This requires $1/3 \leq q \leq 2/3$.

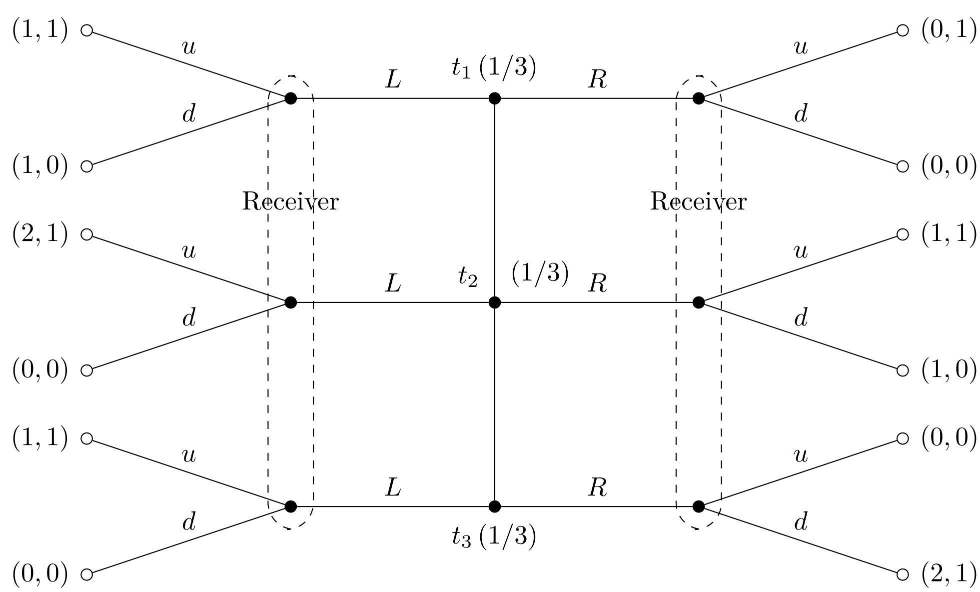
To conclude, $(R, (q, 1-q), p = 1/2)$ for $1/3 \leq q \leq 2/3$ constitutes a mixed strategy PBE.

**Question 3: Gibbons 4.5** Find all the pure-strategy perfect Bayesian equilibria of the following games.

a.




b.



**Solution**

(a) There exist a set of pooling PBE that can be characterized by the following strategy profile:

- Both types of sender plays $R$;
- The receiver plays $u$ if $L$ is observed, plays $d$ if $R$ is observed;

and the receiver's belief: $\Pr(t_1|R) = 1/2$ and $\Pr(t_1|L) \geq 1/3$.

(b) There exist a set of pooling PBE that can be characterized by the following strategy profile:

- All types of sender plays $L$;
- The receiver always plays $u$;

and the receiver's belief: $\Pr(t_1|L) = \Pr(t_2|L) = \Pr(t_3|L) = 1/3$ and $\Pr(t_3|R) \leq 1/2$.

There also exists another PBE that can be characterized by the following strategy profile:

- The type-$t_1$ and type-$t_2$ sender plays $L$, and the type-$t_3$ sender plays $R$;
- The receiver plays $u$ if $L$ is observed, plays $d$ if $R$ is observed;

and the receiver's belief: $\Pr(t_1|L) = \Pr(t_2|L) = 1/2$ and $\Pr(t_3|R) = 1$.