

# Advanced Microeconomics

## Assignment 1

**Due date:** September 25, 2022 (Sunday)

**Submission method:** QQ group

**Grading:** Your assignment will be graded based on your effort, not the accuracy of your answers.

The exercises are embedded in the Chapter 1 lecture notes (red boxes). You are advised to read the relevant sections when you work on the exercises.

The same set of exercises are provided below:

**1.B.3** Show that if  $f : \mathbb{R} \rightarrow \mathbb{R}$  is a strictly increasing function and  $u : X \rightarrow \mathbb{R}$  is a utility function representing preference relation  $\succsim$ , then the function  $v : X \rightarrow \mathbb{R}$  defined by  $v(x) = f(u(x))$  is also a utility function representing preference relation  $\succsim$ .

**1.C.1** Consider the choice structure  $(\mathcal{B}, C(\cdot))$  with  $\mathcal{B} = (\{x, y\}, \{x, y, z\})$  and  $C(\{x, y\}) = \{x\}$ . Show that if  $(\mathcal{B}, C(\cdot))$  satisfies W.A.R.P, then we must have  $C(\{x, y, z\}) = \{x\}$ ,  $\{z\}$ , or  $\{x, z\}$ .

**1.C.2** Show that W.A.R.P (Definition 1.C.1) is equivalent to the following property holding:

Suppose that  $B, B' \in \mathcal{B}$ , that  $x, y \in B$ , and that  $x, y \in B'$ . Then if  $x \in C(B)$  and  $y \in C(B')$ , we must have  $\{x, y\} \subset C(B)$  and  $\{x, y\} \subset C(B')$ .

**1.D.2** Show that if  $X$  is **finite**, then any rational preference relation generates a nonempty choice rule; that is,  $C(B) \neq \emptyset$  for any  $B \subset X$  with  $B \neq \emptyset$ . [hint: utilize the result of Remark 1.]

**1.D.3** Let  $X = \{x, y, z\}$ , and consider the choice structure  $(\mathcal{B}, C(\cdot))$  with

$$\mathcal{B} = \{\{x, y\}, \{y, z\}, \{x, z\}, \{x, y, z\}\}$$

and  $C(\{x, y\}) = \{x\}$ ,  $C(\{y, z\}) = \{y\}$ , and  $C(\{x, z\}) = \{z\}$ , as in Example 1.D.1. Show that  $(\mathcal{B}, C(\cdot))$  must violate W.A.R.P.

**Additional Exercise** Can you think of an example in which the preference relation is transitive but not complete?