Advanced Microeconomics

Assignment 1 Solution

1.B.3 Show that if $f : \mathbb{R} \to \mathbb{R}$ is a strictly increasing funciton and $u : X \to \mathbb{R}$ is a utility function representing preference relation $\succsim$, then the function $v : X \to \mathbb{R}$ defined by $v(x) = f(u(x))$ is also a utility function representing preference relation $\succsim$.

Solution. *We show that* $\forall x, y \in X$, *we have* $x \succsim y$ *iff* $v(x) \geq v(y)$.

Since $u(\cdot)$ *is a utility function representing the preference relation* $\succsim$, *we have*

$$x \succsim y \Leftrightarrow u(x) \geq u(y) \tag{1}$$

As $f(\cdot)$ *is strictly increasing,*

$$u(x) \geq u(y) \Leftrightarrow f(u(x)) \geq f(u(y)) \Leftrightarrow v(x) \geq v(y) \tag{2}$$

From (1) and (2),

$$x \succsim y \Leftrightarrow v(x) \geq v(y).$$

1.C.1 Consider the choice structure $(\mathscr{B}, C(\cdot))$ with $\mathscr{B} = \{\{x, y\}, \{x, y, z\}\}$ and $C(\{x, y\}) = \{x\}$. Show that if $(\mathscr{B}, C(\cdot))$ satisfies the weak axiom, then we must have $C(\{x, y, z\}) = \{x\}, = \{z\}$, or $= \{x, z\}$.

Solution. *We prove by contradiction.*

Suppose the conclusion fails to hold, then we must have

$$y \in C(\{x, y, z\}).$$

$C(\{x, y\}) = \{x\}$ *implies* $x \in C(\{x, y\})$ *and* $y \notin C(\{x, y\})$.

We apply W.A.R.P: Since for $x, y \in \{x, y, z\}$, *we have* $y \in C(\{x, y, z\})$. *Then, for* $x, y \in \{x, y\}$ *and* $x \in C(\{x, y\})$, *we must have* $y \in C(\{x, y\})$. *This contradicts* $y \notin C(\{x, y\})$.

1.C.2 Show that the weak axiom (Definition 1.C.1) is equivalent to the following property holding: Suppose that $B, B' \in \mathscr{B}$, that $x, y \in B$, and that $x, y \in B'$. Then if $x \in C(B)$ and $y \in C(B')$, we must have $\{x, y\} \subset C(B)$, and $\{x, y\} \subset C(B')$.

Definition 1.C.1. The choice structure $(\mathscr{B},\ C(\cdot))$ satisfies the weak axiom of revealed preference if the following property holds:

If for some $B \in \mathscr{B}$ with $x, y \in B$ we have $x \in C(B)$, then for any $B' \in \mathscr{B}$ with $x, y \in B'$ and $y \in C(B')$, we must also have $x \in C(B')$

Solution. *Suppose first that the weak axiom holds. Since $x, y \in B$ and $x \in C(B)$, then, by weak axiom, $x, y \in B'$ and $y \in C(B')$ implies that $x \in C(B')$. Hence, $\{x, y\} \subset C(B')$. Similarly, since $x, y \in B'$ and $y \in C(B')$, then $x, y \in B$ and $x \in C(B)$ would imply that $y \in C(B)$ and hence $\{x, y\} \subset C(B)$.*

Next, suppose the property in the question holds, we want to show that if $B \in \mathscr{B}, x, y \in B$ and $x \in C(B)$, then for $B' \in \mathscr{B}, x, y \in B', y \in C(B')$, we must have $x \in C(B')$. This is immediate since the property implies $\{x, y\} \subset C(B')$.

1.D.2 Show that if X is finite, then any rational preference relation generates a nonempty choice rule; that is, $C(B) \neq \varnothing$ for any $B \subset X$ with $B \neq \varnothing$.

Solution. *By Remark 1, there exists utility function $u(\cdot)$ that represents $\succsim$. Since X is finite, for any $B \subset X$ with $B \neq \varnothing$, there exists $x \in B$ such that $u(x) \geq u(y)$ for all $y \in B$. Then $x \in C^*(B, \succsim)$ and hence $C^*(B, \succsim) \neq \varnothing$.*

1.D.3 Let $X = \{x, y, z\}$, and consider the choice structure $(\mathscr{B}, C(\cdot))$ with

$$\mathscr{B} = \{\{x, y\}, \{y, z\}, \{x, z\}, \{x, y, z\}\}$$

and $C(\{x, y\}) = \{x\}, C(\{y, z\}) = \{y\}$, and $C(\{x, z\}) = \{z\}$, as in Example 1.D.1. Show that $(\mathscr{B}, C(\cdot))$ must violate the weak axiom.

Solution. *If $x \in \{x, y, z\}$, since $x, z \in \{x, y, z\}$, and $x, z \in \{x, z\}$, $z \in C(\{x, z\})$, (implied by $C(\{x, z\}) = \{z\}$), then by W.A.R.P, $x \in C(\{x, z\})$, which contradicts $C(\{x, z\}) = \{z\}$.*

Similarly, $y \in \{x, y, z\}$ contradicts $C(\{x, y\}) = \{x\}$; and $z \in \{x, y, z\}$ contradicts $C(\{y, z\}) = \{y\}$.

Additional Exercise Can you think of an example in which the preference relation is transitive but not complete?

Solution. *Let* $X = \mathbb{R}^2$. *Consider the preference relation: for all* $x, y \in X$, $x \succsim y$ *whenever* $x \geq y$, *that is, whenever* $x_1 \geq y_1$ *and* $x_2 \geq y_2$.

1. *This preference relation is transitive: Let* $x, y, z \in \mathbb{R}^2$. *Suppose* $x \succsim y$ *and* $y \succsim z$. *Then,* $x_1 \geq y_1$ *and* $x_2 \geq y_2$; $y_1 \geq z_1$ *and* $y_2 \geq z_2$. *This implies* $x_1 \geq z_1$ *and* $x_2 \geq z_2$. *Therefore,* $x \succsim z$.

2. *This preference relation is not complete: For* $x = (1, 0)$ *and* $y = (0, 1)$, *we have* $x \not\succsim y$ *and* $y \not\succsim x$.

Remark. The above preference relation is just an example. There are many other preference relations that are transitive but not complete.