Advanced Microeconomics

Assignment 2

Due date: October 23, 2022 (Sunday)

Submission method: QQ group

Grading: Your assignment will be graded based on your effort, not the accuracy of your answers.

The exercises are embedded in the Chapter 2 lecture notes (red boxes). You are advised to read the relevant sections when you work on the exercises.

The same set of exercises are provided below:

2.D.2 A consumer consumes one consumption good x and hours of leisure h. The price of the consumption good is p, and the consumer can work at a wage rate of $s = 1$. What is the consumer's Walrasian budget set?

2.E.1 Suppose $L = 3$, and consider the demand function $x(p, w)$ defined by

$$x_1(p, w) = \frac{p_2}{p_1 + p_2 + p_3} \frac{w}{p_1}$$
$$x_2(p, w) = \frac{p_3}{p_1 + p_2 + p_3} \frac{w}{p_2}$$
$$x_3(p, w) = \frac{\beta p_1}{p_1 + p_2 + p_3} \frac{w}{p_3}$$

Does this demand function satisfy homogeneity of degree zero and Walras' law when $\beta = 1$? What about when $\beta \in (0, 1)$?

2.E.3 Use Proposition 2.E.1 to 2.E.3 to show that $p \cdot D_p x(p, w) p = -w$.

2.E.5 Suppose that $x(p, w)$ is a demand function which is homogeneous of degree one with respect to w and satisfies Walras' law and homogeneity of degree zero. Suppose also that all the cross-price effects are zero, that is $\partial x_l(p, w) / \partial p_k = 0$ whenever $k \neq l$.

Show that this implies that for every l, $x_l(p,w) = \alpha_l w/p_l$, where $\alpha_l > 0$ is a constant independent of (p,w).

2.E.7 A consumer in a two-good economy has a demand function $x(p,w)$ that satisfies Walras' law. His demand function for the first good is $x_1(p,w) = \alpha w/p_1$. Derive his demand function for the second good. Is his demand function homogeneous of degree zero?

2.E.8 Show that the elasticity of demand for good l with respect to price p_k, $\varepsilon_{lk}(p,w)$, can be written as $\varepsilon_{lk}(p,w) = d\ln(x_l(p,w))/d\ln(p_k)$, where $\ln(\cdot)$ is the natural logarithm function. Derive a similar expression for $\varepsilon_{lw}(p,w)$. Conclude that if we estimate the parameters $(\alpha_0, \alpha_1, \alpha_2, \gamma)$ of the equation $\ln(x_l(p,w)) = \alpha_0 + \alpha_1 \ln p_1 + \alpha_2 \ln p_2 + \gamma \ln w$, these parameter estimates provide us with estimates of the elasticities $\varepsilon_{l1}(p,w)$, $\varepsilon_{l2}(p,w)$, and $\varepsilon_{lw}(p,w)$.

2.F.11 Show that for $L = 2$, $S(p,w)$ is always symmetric. [Hint: Use Proposition 2.F.3.]

2.F.17 In an L-commodity world, a consumer's Walrasian demand function is

$$x_k(p,w) = \frac{w}{\sum_{l=1}^{L} p_l} \text{ for } k = 1, ..., L.$$

(a) In this demand function homogeneous of degree zero in (p,w)?

(b) Does it satisfy Walras' law?

(c) Does it satisfy the weak axiom?

(d) Compute the Slutsky substitution matrix for this demand function. Is it negative semidefinite? Negative definite? Symmetric?