Chapter 1. Preference and Choice

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1.A. Introduction

Two approaches to modeling individual choice behavior:

1. Preference-based Approach: preference as primative (rationality axioms) $\Longrightarrow$ consequences on choices

2. Choice-based Approach: choice behavior as primative (axioms on behavior)

1.B. Preference Relations

X: Set of Alternatives.

- For example, if Alice just graduated from Wuhan University majoring in economics, then her set of alternatives is: $X = \{$go to graduate school and study economics, go to a Big-4 firm, go to work for the government, ..., run a small business$\}$.

We use capital letters (like X and B) for a set of alternatives, small letters (like x and y) for a specific choice alternative.

Defining Preference Relations

Denote by $\succsim$ *the preference relation* defined on the set X, allowing the comparison of any x and y in X.

- $x \succsim y$: pronounced as "x is preferred to y" or "x is at least as good as y." The first usage is more common.

- *Strict preference* $\succ$: $x \succ y \iff x \succsim y$ but not $y \succsim x$ (i.e., $y \not\succsim x$) ("x is strictly preferred to y.")

- *Indifference* $\sim$: $x \sim y \iff x \succsim y$ and $y \succsim x$ ("x is indifferent to y.")

Rational Preference

Not all preference relations make sense.

For example, consider Alice's preference:

- "Hot and Dry Noodles" $\succ$ "Doupi" (dòu pí)
- "Doupi" $\succ$ "Xiaolongbao" (xiǎo lóng bāo)
- "Xiaolongbao" $\succ$ "Hot and Dry Noodles"

Alice must have a hard time choosing her breakfast from

$X = \{$Hot and Dry Noodles, Doupi, Xiaolongbao$\}$.

Rational Preference

Definition 1.B.1 (Rational preference)**.** The preference relation $\succsim$ is **rational** if it possesses these two properties:

(i) Completeness: $\forall x, y \in X$, $x \succsim y$ or $y \succsim x$. (rules out $x \not\succsim y$ and $y \not\succsim x$)

(ii) Transitivity: $\forall x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

Rational Preference

Question. In the example above, which property does Alice's preference relation violate?

Rational Preference

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Answer: Transitivity.

Proof by contradiction. The first two bullet points implies

- "Hot and Dry Noodles" $\succsim$ "Doupi" (dòu pí)
- "Doupi" $\succsim$ "Xiaolongbao" (xiǎo lóng bāo)

by transitivity, "Hot and Dry Noodles" $\succsim$ "Xiaolongbao".

This contradicts "Xiaolongbao" $\succ$ "Hot and Dry Noodles".

Rational Preference

Exercise. *Can you think of an example in which the preference relation is transitive but not complete?*

Implications of Rational Preference on $\succ$ and $\sim$

The following propositions follow from the definition of *rational preference*.

Proposition 1.B.1. *If $\succsim$ is rational, then:*

(i) $\succ$ is both irreflexive ($x \succ x$ never holds) and transitive.

(ii) $\sim$ is reflexive ($x \sim x$), transitive and symmetric (if $x \sim y$, then $y \sim x$).

(iii) if $x \succ y \succsim z$, then $x \succ z$. (slightly stronger than transitivity in (i))

Utility Functions

Definition 1.B.2. A function $u : X \to \mathbb{R}$ is a utility function representing preference relation $\succsim$ if

$$x \succsim y \iff u(x) \geq u(y) \text{ for all } x, y \in X. \tag{1}$$

The utility function is nothing but assigning each choice x with a number $u(x)$. Obviously, the function u satisfying Condition (1) is not unique.

Example. $u(x) \geq u(y) \iff \alpha u(x) \geq \alpha u(y)$ for all $\alpha > 0$.

Utility Functions

Exercise. *Show that if* $f : \mathbb{R} \to \mathbb{R}$ *is a **strictly increasing** function and* $u : X \to \mathbb{R}$ *is a utility function representing preference relation* $\succsim$*, then the function* $v : X \to \mathbb{R}$ *defined by* $v(x) = f(u(x))$ *is also a utility function representing preference relation* $\succsim$*.*

Utility Functions

Question. When can a preference relation be represented by a utility function?

Utility Functions

> *Question.* When can a preference relation be represented by a utility function?

Answer: Only if the preference relation is rational. See the next proposition.

Utility Functions

Proposition 1.B.2. *If the preference relation* $\succsim$ *can be represented by a utility function (i.e.* $\exists u(\cdot)$ *s.t.* $u(x) \geq u(y)$ *iff* $x \succsim y$*), then* $\succsim$ *is rational (i.e. complete & transitive).*

Utility Functions

Question. If $\succsim$ is rational, does there exist a utility function u representing $\succsim$?

Utility Functions

Question. If $\succsim$ is rational, does there exist a utility function u representing $\succsim$?

Answer: Not always. Rationality is just a necessary condition for the existence of a utility representation, but not sufficient. See the counterexample below.

Lexicographic Preference

Definition (Lexicographic Preference)**.** Let $X = \mathbb{R}^2$. The preference relation $\succsim$ is a *lexicographic preference* if for all $x, y \in X$, $x \succsim y$ whenever (i) $x_1 > y_1$ or (ii) $x_1 = y_1$ and $x_2 \geq y_2$.

Claim. The lexicographic preference on $\mathbb{R}^2$ do *not* have a utility representation.

Example of Lexicographic Preference

Alice is considering buying a new phone. The relevant attributes include brand name, price, CPU, and so on. For simplicity, suppose Alice only cares about the price and the brand (Apple or Huawei) Alice's first priority is the price. (Of course, Alice prefers low price to high price.) At the same price, Alice prefers an iPhone to a Huawei Phone. For Example,

$$(5000, \text{Huawei}) \succ (8000, \text{Apple}) \succ (8000, \text{Huawei}).$$

Utility Function

Remark. If X is **finite** and $\succsim$ is a rational preference relation on X, then there is a utility function $u : X \to R$ that represents $\succsim$.

1.C. Choice Rules

A *choice structure* $(\mathscr{B}, C(\cdot))$ consists of two ingredients:

(i) $\mathscr{B}$ is a family (a set) of nonempty subsets of X: that is, every $B \in \mathscr{B}$ is a set $B \subset X$.

- In consumer theory, B are budget sets.
- $\mathscr{B}$ needs NOT to include all possible subsets of X.

(ii) $C(\cdot)$ is a choice rule that assigns a nonempty subset of chosen elements $C(B) \subset B$ for every $B \in \mathscr{B}$.

- $C(B)$ is a set of *acceptable alternatives*.

Choice Rules

Example 1.C.1. $X = \{x, y, z\}, \mathscr{B} = \{\{x, y\}, \{x, y, z\}\}$

Choice Structure 1 $(\mathscr{B}, C_1(\cdot))$:

$$C_1(\{x, y\}) = \{x\}, C_1(\{x, y, z\}) = \{x\}$$

Choice Structure 2 $(\mathscr{B}, C_2(\cdot))$:

$$C_2(\{x, y\}) = \{x\}, C_2(\{x, y, z\}) = \{x, y\}$$

Under $(\mathscr{B}, C_2(\cdot))$, y is acceptable only if z is available.

Go to Example 1.C.2

Choice Rules

You might find the choice structure 2 unreasonable.

Consider the following conversation.

Waiter:	Coffee or Tea?
Customer:	Coffee, please.
Waiter:	Sure. Oh sorry, actually we also serve coke. Do you want some coke?
Customer:	*Since coke is available, I'd prefer tea rather than coffee.*

Weak Axiom of Revealed Preference (W.A.R.P)

Definition 1.C.1. The choice structure $(\mathscr{B}, C(\cdot))$ satisfies the weak axiom of revealed preference (W.A.R.P) if the following property holds:

If for some $B \in \mathscr{B}$ with $x, y \in B$ we have $x \in C(B)$, then for any $B' \in \mathscr{B}$ with $x, y \in B'$ and $y \in C(B')$, we must also have $x \in C(B')$.

Weak Axiom of Revealed Preference (W.A.R.P)

In the last example, $(\mathscr{B}, C_2(\cdot))$ violates W.A.R.P since

$y \in C_2(\{x, y, z\})$, $x, y \in \{x, y\}$, $x \in C_2(\{x, y\})$ but

$y \notin C_2(\{x, y\})$.

[Think of $\{x, y, z\}$ as B and $\{x, y\}$ as B' in Definition 1.C.1.]

IDEA: Agent's choice between x and y should not be affected

by irrelevant options/alternatives.

Revealed Preference: Preference inferred from/ revealed through Choice

Definition 1.C.2. Given a choice structure $(\mathscr{B}, C(\cdot))$, the **revealed preference relation** $\succsim^*$ is defined by

$$x \succsim^* y \iff \exists B \in \mathscr{B} \text{ s.t. } x, y \in B \text{ and } x \in C(B).$$

$x \succsim^* y$ reads "x is revealed at least as good as y"

Revealed Preference

- $x \succ^* y$:

  $\exists B \in \mathscr{B}$ s.t. $x, y \in B$ and $x \in C(B),$ and $y \notin C(B)$. (

  "x is revealed preferred to y")

- $\succsim^*$ needs not to be complete or transitive.

- "Revealed preference" is defined reference to B.

  (Compare with "preference")

- **Restatement of W.A.R.P**: If $x \succsim^* y$, then $y \not\succ^* x$.

  (only imposed on $B \in \mathscr{B}$)

Example 1.C.2. Recall Example 1.C.1.

$(\mathscr{B}, C_1(\cdot))$: $x \succ^* y$ and $x \succ^* y$, $x \succ^* z$

$(\mathscr{B}, C_2(\cdot))$: $x \succ^* y$ and $y \succsim^* x \implies$ contradicts W.A.R.P

Useful alternative statements of W.A.R.P

Restatement of W.A.R.P 1. *$x, y \in B$, $x \in C(B)$, $y \in C(B')$ & $x \notin C(B')$, then $x \notin B'$.*

Proof. Proof by contradiction. If $x \in B'$ & $y \in C(B')$, W.A.R.P $\implies$ $x \in C(B')$. $\square$

Restatement of W.A.R.P 2. *Suppose that $B, B' \in \mathscr{B}$, that $x, y \in B$, and that $x, y \in B'$. Then if $x \in C(B)$ and $y \in C(B')$, we must have $\{x, y\} \subset C(B)$ and $\{x, y\} \subset C(B')$.*

The proof is left as an exercise.

1.D. Relationship between Preference Relations & Choice Rules

More precisely, we want to know the relationship between *rational preference* and *W.A.R.P*.

(i) Does Rational Preference imply W.A.R.P?

(ii) Does W.A.R.P imply Rational Preference?

1.D. Relationship between Preference Relations & Choice Rules

More precisely, we want to know the relationship between *rational preference* and *W.A.R.P*.

(i) Does Rational Preference imply W.A.R.P? (Yes)

(ii) Does W.A.R.P imply Rational Preference? (Maybe)

Preference Generated Choice Structure

Consider rational preference $\succsim$ on X.

Define: $C^*(B, \succsim) = \{x \in B\colon x \succsim y$ for every $y \in B\}$

- Elements of $C^*(B, \succsim)$ are DM's most preferred alternatives in B.
- Assumption: $C^*(B, \succsim)$ is nonempty for all $B \in \mathscr{B}$.

Remark. If X is **finite**, then any rational preference relation generates a nonempty choice rule.

The proof is left as an exercise.

Preference Generated Choice Structure

We say that the preference $\succsim$ *generates* the choice structure $(\mathscr{B}, C^*(\cdot, \succsim))$.

Proposition 1.D.1. *Suppose $\succsim$ is a rational preference relation. Then the choice structure generated by $\succsim$, $(\mathscr{B}, C^*(\cdot, \succsim))$ satisfies W.A.R.P.*

Preference Generated Choice Structure

Definition 1.D.1. Given a choice structure $(\mathscr{B}, C(\cdot))$, we say that the rational preference relation $\succsim$ rationalizes $C(\cdot)$ relative to $\mathscr{B}$ if $C(B) = C^*(B, \succsim)$ for all $B \in \mathscr{B}$, that is, if $\succsim$ generates the choice structure $(\mathscr{B}, C(\cdot))$.

Preference Generated Choice Structure

1. If a rational preference relation rationalizes the choice rule, we can interpret the DM's choices as if she were a preference maximizer.

2. In general, there may be more than one rationalizing preference relation $\succsim$ for a given choice structure $(\mathscr{B}, C(\cdot))$.

   Example. $X = \{x, y\}, \mathscr{B} = \{\{x\}, \{y\}\}$, $C(\{x\}) = \{x\}, C(\{y\}) = \{y\}$.

Preference Generated Choice Structure

Example 1.D.1. $X = \{x, y, z\}$,

$\mathscr{B} = \{\{x, y\}, \{y, z\}, \{x, z\}\}$[1],

$C(\{x, y\}) = \{x\}, C(\{y, z\}) = \{y\}, C(\{x, z\}) = \{z\}$.

This choice structure satisfies the W.A.R.P.

However, it cannot be rationalized by a rational preference.

Remark. W.A.R.P is defined by $\mathscr{B}$. And the choice is not challenged by having to choose from $\{x, y, z\}$.

[1] $\{x, y, z\}$ is not empirically relevant.

Preference Generated Choice Structure

Proposition 1.D.2. *If $(\mathscr{B}, C(\cdot))$ is a choice structure such that*

(i) the W.A.R.P is satisfied, [$x \succsim^* y$, then $y \not\succ^* x$]

(ii) $\mathscr{B}$ includes all subsets of X of up to three elements,

then $\exists$ rational $\succsim$ that rationalizes $C(\cdot)$ relative to $\mathscr{B}$, i.e.,

$$C(B) = C^*(B, \succsim), \forall B \in \mathscr{B}.$$

Furthermore, this rational preference relation is unique.

Summary of Chapter 1

- Preference relation $\succsim$ is binary relation on choice set X.

- $\succsim$ is rational if Completeness & Transitivity.

- Choice function $C(\cdot)$ is defined on $\mathscr{B}$, NOT on X.

  (Assumptions: W.A.R.P & $C(\cdot) \neq \varnothing$)

- Rational Preference implies W.A.R.P.

  But for W.A.R.P to imply Rational Preference, it requires $C(\cdot) \neq \varnothing$ and that $\mathscr{B}$ includes all 2 & 3-element subsets of X.

Comments on the Standard Models

- Incomplete data about choice
  - We only observe limited subsets of X.
  - We may probably only observe 1 element $x \in C(A)$.

Comments on the Standard Models

- $\mathscr{B}$ does not include all subsets of X of up to 3 elements
  - WARP does NOT imply the existence of a rational preference. (Example 1.D.1)
  - Simple Generalized Axiom of Revealed Preference (SGARP): $x^1 \succsim^* x^2, ..., x^{n-1} \succsim^* x^n \implies x^n \not\succ^* x^1$

Comments on the Standard Models

- We may probably only observe 1 element $x \in C(A)$.
  - WARP has zero empirical implications.
  - In consumer demand theory, may assume that $C(B)$ is a singleton for some good B.

Comments on the Standard Models

- Indecisiveness: sometimes when asked to rank x and y, one is just unable to decide.
  - Add the possibility "I can't rank them" ($\succsim$ is not complete)
- No utility representation in this case as set $\mathbb{R}$ is *totally ordered.*

Comments on the Standard Models

- Framing: the way an object is presented may change how a consumer perceives it and therefore affect the choices she makes. Tversky and Kahneman (1981)
  - A: save 400 out of 600 ppl;

    B: save no one with probability $1/3$, and save all 600 with probability $2/3$.
  - X: 200 out of 600 ppl will die with certainty;

    Y: probability $2/3$ that no one will die and probability $1/3$ that all 600 will die.

More on Framing

The Economist has three subscription options:

1. Internet-only subscription for $59.
2. Print-only subscription for $125.
3. Print-and-Internet subscription for $125.

More on Framing

An experiment is run on 100 students.

	Experiment 1
Internet-only	16
Print-only	0
Print-and-Internet	84

What do you expect would happen if the seemingly irrelevant option “Print-only” is removed?

More on Framing

They choose differently!

	Experiment 1	Experiment 2
Internet-only	16	68
Print-only	0	Not Available
Print-and-Internet	84	32