Dynamic Optimization

Assignment 2

Due date: April 18, 2022

Submission method: QQ group

There are 3 questions in total. Please present your work in a neat and organized manner.

Question 1: Exercise 8.2 Minimization

Part I Develop second-order sufficient conditions for the unconstrained **minimization** problem.

You may use the definitions and the determinantal tests of *positive (semi-)definite* matrix in the Lecture Notes directly. If you want, you could also develop the determinantal tests from the tests for *negative (semi-)definite* matrix. In your derivation, you will find the following result useful: $\det(-A) = (-1)^n \det(A)$ for an $n \times n$ matrix A.

Part 2 Use Theorem 8.4 in the Lecture Notes to develope the second-order sufficient condition for the constrained **minimization** problem. (Hint: $\det(-A) = (-1)^n \det(A)$ for an $n \times n$ matrix A.)

Question 2 Consider the firm's cost minimization problem:

$$\begin{aligned} &\min_z w \cdot z \\ \text{s.t. } &f(z) \geq q, \end{aligned}$$

where w is the vector of input prices, z is a vector of input quantities, q is the target output level, and f is the production function. Let $c(w, q)$ be the minimized cost. The corresponding *conditional factor demand function* $z(w, q)$ is the vector of quantities that solves the cost minimization problem.

Prove the following claims:

(i) $c(w, q)$ is a concave function of w.

(ii) If f is strictly quasi-concave, $z(w, q)$ is unique.

(iii) Suppose $c(w, q)$ is differentiable with respect to w, then $z(w, q) = c_w(w, q)$. This result is called *Shepard's lemma*.

(iv) $z_w(w, q)$ is symmetric and negative semi-definite. (Hint: use (i) and (iii).)

(v) If f is concave, then $c(w, q)$ is a convex function of q.

Question 3 Consider the following maximization problem with 4 variables and 2 constraints:

$$\begin{aligned} &\max_{x,y,z,w} F(x, y, z, w) \equiv 2x + y - z \\ \text{s.t. } &G^1(x, y, z, w) \equiv y - 4w - 5z = 0 \\ &G^2(x, y, z, w) \equiv x^2 + z^2 + w^2 = 9 \end{aligned}$$

Part I Use the first-order necessary condition to solve for the stationary points.

Part II Use the second-order sufficient condition to determine which stationary point is a local maximum.