# Dynamic Optimization

# Assignment 3

**Due date:** May 6, 2022

**Submission method:** QQ group

**There are 4 questions in total. Please present your work in a neat and organized manner.**

**Question 1: Exercise 9.1 (Taxation of Risky Income)** Consider the model of one safe and one risky asset. Now, we introduce taxation of interest income on the risky asset (with deduction allowed for losses) at the rate $\tau \in (0,1)$. Thus, the final random wealth is

$$W = W_0 + (1-\tau)xr.$$

1. Show that the first-order condition for an interior optimum $x$ is

$$\int_{\underline{r}}^{\overline{r}} rU'(W_0 + (1-\tau)xr)f(r)\mathrm{d}r = 0.$$

2. Deduce that if $\tau$ changes, the optimum $x$ changes keeping $(1-\tau)x$ constant. Therefore, if the tax rate on risky income increases, so does the amount of wealth held in the risky asset. Suggest an economic intuition.

**Question 2 Exercise 9.2 (Saving with Uncertainty)** A consumer lives for two periods. Income in period 1 is sure and equal to $Y_1$. The income $Y_2$ in period 2 can be random. If he saves $S$ from his period-1 income, he gets total return (principal plus interest) of $rS$ in period 2, where $r$ can be random. His objective is to maximize the expected present value of the utility of consumption in the two periods:

$$U(Y_1 - S) + \delta\,\mathbb{E}[U(Y_2 + rS)].$$

where $U' > 0$ and $U'' < 0$.

1. Write down the first- and second-order conditions. Show that as $Y_1$ increases, $S$ also increases but at a smaller rate, that is, the marginal propensity to save lies between 0 and 1.

2. Next, suppose that $Y_2$ is sure but $r$ is random, and examine the effect of an increase in $Y_2$.

3. Finally, suppose $r$ is sure but $Y_2$ is random, and examine the effect of an increase in $r$.

**Question 3:** Reconsider the insurance problem. Now we assume that the insurance policy is **not** acturially fair. The problem is elaborated below.

Suppose $Y_1 < Y_2$, which means the first state entails some loss relative to the second. $Y_1$ occurs with probability $p$ and $Y_2$ with probability $(1-p)$. Insurance company sells an insurance at constant rate $q$ per \$1 coverage. That is, if the individual purchases $x$ shares of insurance, the insurance company would pay the individual $X = x/q$ when the bad outcome (state 1) occurs. The insurance policy is **not** acturially fair, i.e., $x - pX > 0$[1] $\iff q > p$.

Show that when the insurance policy is not acturially fair, the risk-averse individual will not be fully insured, i.e., $Y_1 - x^* + x^*/q < Y_2 - x^*$.

**Question 4: Screening** Consider the contracting problem between a seller (he) and a buyer (she). The buyer's utility function takes the form:

$$u(q, T, \theta_i) = \theta_i v(q) - T, \tag{1}$$

where $q$ is the quantity consumed, $T$ is the payment to the seller, and $\theta_i = \{\theta_H, \theta_L\}$, where $\theta_H > \theta_L > 0$, is the buyer's type. The buyer is privately informed of her own type. Assume that $v$ is strictly increasing and strictly concave. Assume also that $v'(0) > c/\theta_H$. The buyer's outside option is 0.

[1]Insurance company makes some profit.

The seller has constant marginal production cost $c > 0$. The profit to the seller is

$$T - cq \tag{2}$$

when he sells quantity $q$ and obtains payment $T$.

1. Assume that the buyer's type is observable by the seller.
   a) Write out the seller's profit maximization problem.
   b) Solve for the optimal contract.
2. Assume that the buyer's type is not observable by the seller. Before contracting, the seller's estimate of the probability of the buyer being Type $\theta_H$ is $\beta_H$, and $\theta_L$ is $\beta_L = (1 - \beta_H)$. The seller can offer different contracts $(q_H, T_H)$ and $(q_L, T_L)$ aiming at Type $\theta_H$ and Type $\theta_L$ respectively.
   a) Write out the seller's profit maximization problem.
   b) Solve for the optimal menu of contracts $(q_H, T_H)$ and $(q_L, T_L)$.